Section 1.5 Parametric Relations and Inverses

A natural way to define functions is to define both elements of the ordered pair (x,y) in terms of another variable t (called a parameter).

Example 1) Consider the set of all ordered pairs (x,y) defined by the equations

$x = t + 1$

$y = t^2 + 2t$ where t is any real number.

a) Find the points determined by t= -3,-2,-1,0,1,2,3

b) Find an algebraic relationship between x and y (this will eliminate the parameter). Is y a function of x?

c) Graph the relation (by hand) in the (x,y) plane.

a)

t	x	y	
-3	-2	3	(-2,3)
-2	-1	0	(-1,0)
-1	0	-1	(0,-1)
0	1	0	(1,0)
1	2	3	(2,3)
2	3	8	(3,8)
3	4	15	(4,15)

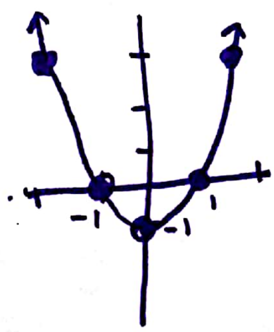
b) $t = x - 1$

$y = (x-1)^2 + 2(x-1)$

$= x^2 - 2x + 1 + 2x - 2$

$y = x^2 - 1$

yes y is a fnc of x



Inverse Relations and Inverse Functions

The ordered pair (a,b) is in a relation if and only if the ordered pair (b,a) is in the inverse relation.

Will the inverse of a function also be a function?

For a graph, the HORIZONTAL LINE TEST can help determine whether or not the inverse is also a function. The inverse of a relation is a function if and only if each horizontal line intersects the graph of the original relation in at most one point.

One-to-one function: Each x is paired with exactly one y and each y is paired with exactly one x.

If f is one-to-one with domain D and range R then the inverse of f, f^{-1} is the function with domain R and range D defined by

$$f^{-1}(b) = a \text{ if and only if } f(a) = b$$

input output

(x,y) on f

(y,x) on f^{-1}

Graph f and f^{-1}

f^{-1} is f reflected in line y = x

Examples

a) Find an equation for $f^{-1}(x)$ if $f(x) = \dfrac{x}{x+1}$

change f(x) to y and solve for the other variable x.

Inverse can be found by undoing.

$y = \dfrac{x}{x+1}$ Solve for x

$y(x+1) = x$

$xy + y = x$

$xy - x = -y$

$x(y-1) = -y$

$x = \dfrac{-y}{y-1}$

$\to x = \dfrac{y}{1-y}$

$f^{-1}(y) = \dfrac{y}{1-y}$ or $f^{-1}(x) = \dfrac{x}{1-x}$

b) Find an equation for $f^{-1}(x)$ if $f(x) = \sqrt{x+3}$

State any restrictions on the domain of f and f^{-1}

$y^2 = \sqrt{x+3}^{\,2}$ $x \geq -3$ D of f: $[-3, \infty)$

R of f: $[0, \infty)$

$y^2 = x + 3$

D of f^{-1}: $[0, \infty)$

$y^2 - 3 = x$

$f^{-1}(y) = y^2 - 3$

$f^{-1}(x) = x^2 - 3$

For inverse
$x \geq 0$
$y \geq -3$

c) Show algebraically that the following functions are inverses by <u>the Inverse Composition Rule.</u>

$f(g(x)) = x$ and $g(f(x)) = x$

Show every step neatly. You may only cancel one term per step.

$$f(x) = 3x - 2 \text{ and } g(x) = \frac{x+2}{3}$$

$$f(g(x)) = f\left(\frac{x+2}{3}\right) = \cancel{3} \cdot \frac{(x+2)}{\cancel{3}} - 2$$

$$= x + \cancel{2} - \cancel{2}$$

$$= x$$

$$g(f(x)) = g(3x-2) = \frac{3x - \cancel{2} + \cancel{2}}{3}$$

$$= \frac{\cancel{3}x}{\cancel{3}}$$

$$= x$$